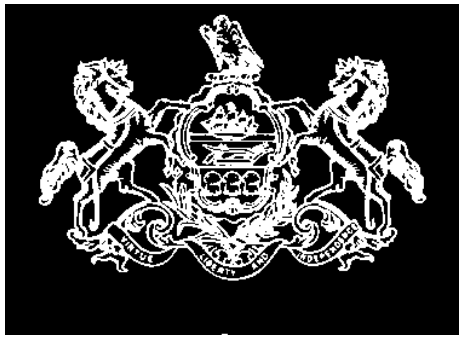
Alternate Academic Content Standards for Mathematics

For Students with the Most Severe Cognitive Disabilities



Pennsylvania Department of Education

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Academic Standards for Mathematics

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II. INTRODUCTION

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… re-interpreted for students with the most severe cognitive disabilities

The Alternate Mathematics Standards describe what students with the most severe cognitive disabilities should know and be able to do at four grade level spans (third/fourth, fifth/sixth, seventh/eighth, and eleventh). **The standards for each grade span were derived from the general education content standards for the equivalent grade that appear in the Chapter 4 Academic Standards for Mathematics as approved by the Pennsylvania Board of Education for all students, at the same grade levels as originally listed (the reference number to the original Chapter 4 Standard is provided in parentheses). The third/fourth grade alternate standards link to the third grade Chapter 4 standards; the eleventh grade alternate standards link to the eleventh grade Chapter 4 standards; and so on.** Those Chapter 4 grade-level content standards have been reinterpreted to be appropriate learning targets for students with the most severe cognitive disabilities, and at the same time link the learning goals for these students with those of their non-disabled grade-mates. The alternate academic content standards delineated in this document provide the targets for instruction and student learning essential for success in the environments in and out of school that students with severe disabilities are likely to encounter. Although the standards are not a curriculum or a prescribed series of activities, school programs for students with the most severe cognitive disabilities will use them to develop a local school curriculum that will meet local students' needs.

The Alternate Mathematics Standards define the skills and strategies employed by students with the most severe cognitive disabilities who have attained proficiency in numeracy skills defined very broadly; all teachers who interact with these students will assist them in learning these skills and strategies through multiple classroom situations in all the subject areas. The Alternate Mathematics Standards also provide parents and community members with information about what students with the most severe cognitive disabilities should know and be able to do as they progress through their educational program and at graduation. With a clearly defined target provided by the standards, parents, students, educators and community members become partners in learning success.

NOTE: The system used in this document to reference the standards is as follows.

Standards listed in boldface are the same standards that appear in the Chapter 4 Academic Standards for Mathematics as approved by the Pennsylvania Board of Education for all students, at the same grade levels as originally listed (the reference number to the original Chapter 4 Standard is provided in parentheses).

Standards listed in regular print **are standards that are continued into additional grades** for ongoing instruction for students with the most severe cognitive disabilities.

Specific skills that define the reinterpreted content standard for students with the most severe cognitive disabilities are provided in italics

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2.1. Numbers, Number Systems and Number Relationships
2.2. Computation and Estimation

GRADE 3/4	GRADE 5/6	GRADE 7/8	GRADE 11
Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:			
A. Count using whole numbers (to 10,000) and by 2's, 3's, 5's, 10's, 25's and 100's. (2.1.3.A) **B. Use concrete objects to count, order and group. (2.1.3.G)** **C. Demonstrate an understanding of one-to-one correspondence. (2.1.3.H)** *Rote counts by 1s up to 9* *Rationally counts single set of items by 1s up to 9 with and without teacher pointing* *Rationally counts groups of sets* *Orders sets of items by quantity* *Sorts objects into groups* *Creates sets of items (e.g., each set has one)* *Counts out quantity named up to 9*	A. Count using whole numbers (to 10,000) and by 2's, 3's, 5's, 10's, 25's and 100's. B. Use concrete objects to count, order and group. C. Demonstrate an understanding of one-to-one correspondence *Rote counts by 1s, 2s, and 5s up to 19* *Rationally counts by 1s, 2s, 5s up to 19 with and without teacher pointing; fixed/movable array* *Rationally counts groups of sets* *Orders sets of items by quantity* *Sorts items into groups* *Creates sets of items (e.g., each set has one)* *Counts out quantity named up to 19*	A. Count using whole numbers (to 10,000) and by 2's, 3's, 5's, 10's, 25's and 100's. B. Use concrete objects to count, order and group. C. Demonstrate an understanding of one-to-one correspondence *Rote counts by 1s, 2s, 5s, 10s, 20s, and 25s up to 99* *Rationally counts by 1s, 2s, 5s, 10s, 20s and 25s up to 99; fixed/movable array* *Counts out quantity named up to 99*	A. Count using whole numbers (to 10,000) and by 2's, 3's, 5's, 10's, 25's and 100's B. Use concrete objects to count, order and group. C. Demonstrate an understanding of one-to-one correspondence *Rote counts by 1s,2s, 5s,10s, 20s, 25s, and 100s up to 500* *Rationally counts by 1s , 2s, 5s,10s, 20s, 25s, and 100s up to 500* *Counts out quantity named up to 500*
D. Use whole numbers and fractions to represent quantities. (2.1.3.B) *Identifies numerals (0-9)* *Matches numbers to quantities (0 – 9)*	D. Use whole numbers and fractions to represent quantities. *Identifies numerals (0 – 19)* *Matches numbers to quantities (0 – 19)* *Locates numerals in an array*	D. Use whole numbers and fractions to represent quantities. *Identifies numerals (0 – 99)* *Matches numbers to quantities (0 –99)*	D. Use whole numbers and fractions to represent quantities. *Identifies numerals (0 – 500)* *Matches numbers to quantities (0 – 500)*

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E. Represent equivalent forms of the same number through the use of concrete objects, drawings, word names and symbols. (2.1.3.C) *Matches numerals to quantities (0 – 9)* *Matches sets with identical quantities* *Reads numerals*	**E. Apply number theory concepts to rename a number quantity (e.g., six, 6, $\frac{12}{2}$, 3 × 2, 10 – 4). (2.1.5.B)** *Matches equivalent numerals, sets, notations (0-19)* *Matches numerals to word (3 to three)*	E. Apply number theory concepts to rename a number quantity (e.g., six, 6, $\frac{12}{2}$, 3 × 2, 10 – 4). *Matches equivalent numerals, sets, notations (0-99)* *Matches numerals to word (3 to three)*	E. Apply number theory concepts to rename a number quantity (e.g., six, 6, $\frac{12}{2}$, 3 × 2, 10 – 4). *Matches equivalent numerals, sets, notations (0-500)* *Matches numerals to word (3 to three*
F. Use drawings, diagrams or models to show the concept of fraction as part of a whole. (2.1.3.D) *Identifies object divided into halves*	**F. Use models to represent fractions. (2.1.5.D)** *Identifies objects divided into equal parts*	F. Use models to represent fractions. *Identifies largest or smallest portions* *Identifies diagram showing x partss of whole* *Identifies diagram showing x pieces of whole (halves, fourths)* *Identifies diagram showing sum of two portions (2 halves; 3 fourths)*	F. Use models to represent fractions. *Identifies largest or smallest portions* *Identifies diagram showing x pieces of whole (halves, thirds, fourths, eights)*
G. Count, compare and make change using a collection of coins and one-dollar bills. (2.1.3.E) H. *Differentiates between "money" and non money* *Differentiates between coins and bills* *Identifies currency named (1,5 dollar bill)* *Names currency* *Reads price up to $5/5¢*	G. Count, compare and make change using a collection of coins and one-dollar bills. (up to 9) **H. Apply estimation strategies to a variety of problems including time and money. (2.2.5.E)** *Identifies value of coins/bills* *Differentiates bills* *Counts pennies, one and five dollar bills* *Estimates costs of items (relative and actual)*	G. Count, compare and make change using a collection of coins and one-dollar bills. (up to 99) H. Apply estimation strategies to a variety of problems including time and money. *Identifies value of coins/bills* *Counts bills with mixed denominations* *Count bills and coins with mixed denominations* *Identifies sufficient amounts for purchases* *Estimates cost of items (relative and actual)*	G. Count, compare and make change using a collection of coins and one-dollar bills. (up to 500) H. Apply estimation strategies to a variety of problems including time and money. *Counts bills with mixed denominations* *Counts bills and coins* *Identifies sufficiency for purchases* *Estimates costs of items (relative and actual)*

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I. Apply number patterns (even and odd) and compare values of numbers on the hundred board. (2.1.3.F) *Sequentially orders consecutive numerals and quantities (0 – 9)*	I. Apply number patterns (even and odd) and compare values of numbers on the hundred board. *Sequentially orders consecutive and non consecutive numerals and quantities (0 – 19)*	I. Apply number patterns (even and odd) and compare values of numbers on the hundred boards. Sequentially orders consecutive and non consecutive numerals and quantities (0 – 99)	I. Apply number patterns (even and odd) and compare values of numbers on the hundred board. *Sequentially orders consecutive and non consecutive numerals and quantities (0 – 500)*
J. Apply addition and subtraction in everyday situations using concrete objects. (2.2.3.A) **K. Apply place-value concepts and numeration to counting, ordering and grouping. (2.1.3.I)** *(1-9)* *Matches numerals to quantities* *Names numerals* *Writes numerals* *Identifies first, second, third, etc.* *Orders consecutive and non consecutive numbers*	J. Apply addition and subtraction in everyday situations using concrete objects. K. Apply place-value concepts and numeration to counting, ordering and grouping. *(0-19)* *Matches numerals to quantities* *Names numerals* *Writes numerals* *Identifies first, second, third, etc.* *Orders consecutive and non consecutive numbers*	J. Apply addition and subtraction in everyday situations using concrete objects K. Apply place-value concepts and numeration to counting, ordering and grouping. *(0-99)* *Matches numeral to quantity* *Names numerals* *Writes numerals* *Identifies first, second, third, etc.* *Orders consecutive and non consecutive numbers*	J. Apply addition and subtraction in everyday situations using concrete objects. K. Apply place-value concepts and numeration to counting, ordering and grouping. *(0-500)* *Matches numerals to quantities* *Names numerals* *Writes numerals* *Identifies first, second, third, etc.* *Orders consecutive and non consecutive numbers*
L. Estimate, approximate, round or use exact numbers as appropriate. (2.1.3.J) *Identifies sets with many/few; most/fewest/least* *Identifies subsets with all, some, most, none*	L. Estimate, approximate, round or use exact numbers as appropriate *Identifies sets with many/few; most/fewest/least* *Identifies subsets with all, some, most, none* *Identifies sufficiency for need*	L. Estimate, approximate, round or use exact numbers as appropriate *Identifies sets with many/few; most/fewest/least* *Identifies subsets with all, some, most, none* *Identifies sufficiency for need*	L. Estimate, approximate, round or use exact numbers as appropriate *Identifies sets with many/few; most/fewest/least* *Identifies subsets with all, some, most, none* *Identifies sufficiency for need*
M. Demonstrate knowledge of basic facts in four basic operations. (2.1.3.L) **N. Solve single- and double-digit addition and subtraction problems with regrouping in vertical form. (2.2.3.B)** **O. Demonstrate the concept of multiplication as repeated addition**	M. Demonstrate knowledge of basic facts in four basic operations. N. Solve single- and double-digit addition and subtraction problems with regrouping in vertical form O. Demonstrate the concept of multiplication as repeated addition and arrays. P. Demonstrate the concept of division	M. Demonstrate knowledge of basic facts in four basic operations. N. Solve single- and double-digit addition and subtraction problems with regrouping in vertical form O. Demonstrate the concept of multiplication as repeated addition and arrays. P. Demonstrate the concept of division as	M. Demonstrate knowledge of basic facts in four basic operations. N. Solve single- and double-digit addition and subtraction problems with regrouping in vertical form O. Demonstrate the concept of multiplication as repeated addition and arrays. P. Demonstrate the concept of division as

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and arrays. (2.2.3.C) **P. Demonstrate the concept of division as repeated subtraction and as sharing. (2.2.3.D)** (0-9) *Adds/subtracts by counting* *Sorts items into groups*	as repeated subtraction and as sharing *(0-19*) *Identify symbols +, -, x, ÷* *Writes equations for addition* *Writes equations for subtraction* *Adds 2 – 3 single digit numbers** *Adds single digit to double digit numbers, without renaming; with renaming** *Adds two double digit numbers, without renaming; with renaming** *Adds two three digit numbers, without renaming; with renaming** *Subtracts one digit numbers** *Subtracts one and two digit numbers without renaming; with renaming** *Subtracts two three digit numbers without renaming; with renaming** *Continuously counts /separate sets of items* **With or without calculator* **Q. Develop and apply algorithms to solve word problems (2.2.5.B)**	repeated subtraction and as sharing *(0-99)* *Identify symbols +, -, x, ÷* *Write equations for addition* *Write equations for subtraction* *Adds single digit numbers** *Adds single digit to double digit numbers, without renaming; with renaming** *Adds two double digit numbers, without renaming; with renaming** *Adds two three digit numbers, without renaming; with renaming** *Subtracts one digit numbers** *Subtracts one and two digit numbers without renaming; with renaming** *Subtracts two three digit numbers without renaming; with renaming** *Multiplies numbers** *Divides numbers** *With or without calculator Q. Develop and apply algorithms to solve word problems **F. Estimate amount of tips and discounts using ratios, proportions and percents. 2.2.8.D)** *Calculates tips** *Calculates discounts** *With or without calculator	repeated subtraction and as sharing *(0-500)* *Identify symbols +, -, x, ÷* *Write equations for addition* *Write equations for subtraction* *Adds single digit numbers** *Adds single digit to double digit numbers, without renaming; with renaming** *Adds two double digit numbers, without renaming; with renaming** *Adds two three digit numbers, without renaming; with renaming** *Subtracts one digit numbers* *Subtracts one and two digit numbers without renaming; with renaming** *Subtracts two three digit numbers without renaming; with renaming** *Multiplies numbers** *Divides numbers** **With or without calculator* Q. Develop and apply algorithms to solve word problems *Selects appropriate operation for word problems*

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2.3. Measurement and Estimation

GRADE 3/4	GRADE 5/6	GRADE 7/8	GRADE 11
Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills to:			
A. Compare measurable characteristics of different objects on the same dimensions (e.g., time, temperature, area, length, weight, capacity, perimeter).(2.3.3.A) *Identifies lightest/heaviest, smallest/biggest, shortest/furthest, shortest/tallest, longest/shortest etc.** *Matches items identical based on length, size, volume* * differences are obvious and vary directly	A. Compare measurable characteristics of different objects on the same dimensions (e.g., time, temperature, area, length, weight, capacity, perimeter). *Identifies lightest/heaviest, smallest/biggest, shortest/furthest, shortest/tallest, longest/shortest, a lot/a little, all/some/most, etc.** *Matches items identical based on length, size, volume** * differences less obvious and vary directly	**A.** Compare measurable characteristics of different objects on the same dimensions (e.g., time, temperature, area, length, weight, capacity, perimeter**).** *Identifies lightest/heaviest, smallest/biggest, shortest/furthest, shortest/tallest, longest/shortest, , a lot/a little, all/some/most, etc.** *Matches items to space* *Orders items by weight/size* *Understands meaning of temperatures* *Matches approximate numerals* * differences obvious and vary on multiple dimensions	A. Compare measurable characteristics of different objects on the same dimensions (e.g., time, temperature, area, length, weight, capacity, perimeter). *Identifies lightest/heaviest, smallest/biggest, shortest/furthest, shortest/tallest, longest/shortest, coldest/hottest, a lot/a little, all/some/most, etc.** *Orders items by weight/size* *Sorts by weight* *Understands meaning of temperatures* * differences less obvious vary on multiple dimensions
B. Determine the measurement of objects with non-standard and standard units (e.g., US customary and metric). (2.3.3.B) **C. Determine the appropriate unit of measure. (2.3.3.E)** *Measures in inches with fixed ruler*	**B. Select and use appropriate instruments and units for measuring quantities (e.g., perimeter, volume, area, weight, time, temperature). (2.3.5.A)** **C. Select and use standard tools to measure the size of figures with specified accuracy, including**	**B. Estimate, use and describe measures of distance, rate, perimeter, area. (2.3.8.D)** C. Select and use standard tools to measure the size of figures with specified accuracy, including length, width, perimeter and area.	B. Select and use appropriate units and tools to measure to the degree of accuracy required in particular measurement situations. **C. Demonstrate the ability to produce measures with specified levels of precision (2.3.11.C)**

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Uses objects (blocks) to measure area and volume *Selects appropriate measurement tool for activities of daily living (clock for time; cup for cooking; rulers for length, etc*	**length, width, perimeter and area. (2.3.5.B)** *Selects cup; identifies1 cupmeasure line* *Measures in cups* *Measures to within inch* *Matches measurement tool to need* *Identifies relative length/distance when path direct*	*Measures to within half inch* *Identifies 1/2,1/4 cup measures* *Identifies 1/2,1/4 cup line* *Identifies relative length when path not direct*	*Identifies empty/half full/full items* *Measures to within quarter inch* *Identifies 1/2,1/4 cup measures* *Identifies 1/2,1/4 cup line* *Identifies shortest path between multiple points*
D. Determine and compare elapsed times. 2.3.3.C)	D. Determine and compare elapsed times.	D. Determine and compare elapsed times.	D. Determine and compare elapsed times.
E. Tell time (analog and digital) to the minute. (2.3.3.D) *Identifies clock* *Reads digital times to hour/half hour*	E. Tell time (analog and digital) to the minute. *Identifies clock* *Matches activities and seasons* *Matches analog and digital times to hour* *Reads digital times to quarter hour* *Reads analog time to hour*	E. Tell time (analog and digital) to the minute. *Matches activities and seasons* *Matches activities to time of day* *Matches activities with duration* *Matches analog and digital times to half hour* *Reads digital times to minute* *Reads analog time to quarter hour*	E. Tell time (analog and digital) to the minute. *Matches activities with duration* *Matches activities to time of day* *Identifies duration of activity given start and end time* *Identifies ending time given start time and duration of activity* *Matches analog and digital times to 5 minute* *Reads analog time to 5 minute*
		F. Determine the appropriateness of overestimating or underestimating in computation. (2.2.8.E) *Describes consequences of having too much/ not enough in common situations*	F. Determine the appropriateness of overestimating or underestimating in computation. *Describes consequences of having too much/ not enough in common situations*

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2.6. Statistics and Data Analysis
2.7. Probability and Predictions

GRADE 3/4	GRADE 5/6	GRADE 7/8	GRADE 11
Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills to:			
A. **Answer questions based on data shown on graphs. (2.6.3.A)** B. **Analyze data using the concepts of largest, smallest, most often, least often and middle. (2.7.3.D)** *Identifies value on graph ordered by size and with and without number prompts* *Biggest/smallest* *Most often/least often/middle* **C. Predict and measure the likelihood of events and recognize that the results of an experiment may not match predicted outcomes. (2.7.3.D)**	A. Answer questions based on data shown on graphs**.** **B. Construct simple conclusions based on data. (2.6.5.E)** *Identifies value on graph with and without number prompts* *Biggest/smallest* *Most often/least often/middle* **C.** Predict and measure the likelihood of events and recognize that the results of an experiment may not match predicted outcomes.	A. Answer questions based on data shown on graphs. **B. Compare and contrast different plots of data using values mode, and range. (2.6.8.A)** *Identifies value on graph with and without number prompts* *Biggest/smallest* *Most often/least often/middle* C. Predict and measure the likelihood of events and recognize that the results of an experiment may not match predicted outcomes**.** *Identifies most likely outcome based on data*	A. Answer questions based on data shown on graphs. B, *Identifies value on graph* *Biggest/smallest* *Most often/least often/middle* C. Predict and measure the likelihood of events and recognize that the results of an experiment may not match predicted outcomes**.** *Identifies most/least likely outcome base don data*

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2.8. Algebra and Functions

GRADE 3/4	GRADE 5/6	GRADE 7/8	GRADE 11
Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills to:			
A. **Recognize, describe, extend, create and replicate a variety of patterns including attribute, activity, number and geometric patterns. (2.8.3.A)** *Sorts dissimilar items into groups with or without sample* B. **Describe and interpret the data shown in tables and charts. (2.8.3.H)** *Locates values identified in table of 6 numbers with column or row identified*	A. **Recognize, reproduce, extend, create and describe patterns, sequences and relationships verbally, numerically, symbolically and graphically, using a variety of materials. (2.8.5.A)** *Sorts dissimilar and similar items into groups with or without sample* B. Describe and interpret the data shown in tables and charts. *Locates values identified in table of 8 numbers with column or row identified*	A. Recognize, reproduce, extend, create and describe patterns, sequences and relationships verbally, numerically, symbolically and graphically, using a variety of materials**.** *Sorts similar items into groups with or without sample* B. Describe and interpret the data shown in tables and charts. *Locates values described in table of 10 numbers with column or row identified*	A. Recognize, reproduce, extend, create and describe patterns, sequences and relationships verbally, numerically, symbolically and graphically, using a variety of materials**.** *Sorts similar items into groups with or without sample* B. Describe and interpret the data shown in tables and charts. *Locates value described in table of 12 numbers with or without column or row identified*